

II PUC MID-TERM EXAMINATION, SEPT./OCT.-2026

SUBJECT : MATHEMATICS (35)

Time : 3 Hours

Max Marks : 80

Instructions :

- The question paper consists of 5 parts A, B, C, D and E. Answer all the parts.
- Part A has 15 Multiple choice questions, 5 Fill in the blank questions of 1 mark each.
- Use the graph sheet for the question on linear programming problem in Part-E.
- Write the question number properly as indicated in the question paper.

PART - A

I Answer ALL the multiple choice questions :

15x1=15

- Let $S = \{1, 2, 3\}$, then the number equivalence relations containing $(1, 2)$ is
A) 1 B) 2 C) 3 D) 4
- Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined as $f(x) = \frac{1}{x}, \forall x \in \mathbb{R}$, then f is
A) one-one B) onto C) bijective D) f is not defined
- The value of $\tan^{-1}(1) + \sin^{-1}\left(\frac{-1}{2}\right) + \cos^{-1}\left(\frac{-1}{2}\right)$ is equal to
A) $\frac{3\pi}{4}$ B) $\frac{3\pi}{2}$ C) $\frac{\pi}{4}$ D) $\frac{\pi}{3}$
- Statement 1 : The principal value of $\cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right) = \frac{5\pi}{6}$.
Statement 2 : Principal branch of inverse cosine function is $[0, \pi]$ and $\cos^{-1}(\cos x) = x$, if $x \in [0, \pi]$
A) Statement 1 is true and Statement 2 is false
B) Statement 1 is true and Statement 2 is true, Statement 2 is correct explanation for Statement 1
C) Statement 1 is true and Statement 2 is true, Statement 2 is not a correct explanation for Statement 1
D) Statement 1 is false and Statement 2 is false.
- If a matrix $A = [a_{ij}]_{m \times n}$, Match List with List II

List I	List II
a) Scalar matrix	i) $a_{ij} = 1$, if $i = j$ and $a_{ij} = 0$ if $i \neq j$
b) Diagonal matrix	ii) $a_{ij} = k$ if $i = j$ and $a_{ij} = 0$ if $i \neq j$
c) Identity matrix	iii) $a_{ij} = 0$ if $i \neq j$

Choose the correct answer from the options given below :

- A) a-i, b-ii, c-iii B) a-iii, b-ii, c-i C) a-ii, b-iii, c-i D) a-iii, b-i, c-ii
- If A is a square matrix, then $A - A^T$ is
A) Unit matrix B) Null matrix C) Skew symmetric matrix D) Symmetric matrix
 - If A is invertible matrix of order two then $|AA^{-1}|$ is
a) 1 B) A C) A' D) 1
 - Right hand derivative of $f(x) = |x|$ at $x = 0$ is
A) -1 B) 1 C) 0 D) does not exist (P.T.O.)

- 9) If $y = \cos^{-1}(\sin x)$, then $\frac{dy}{dx}$ is equal to
 A) -1 B) 1 C) $\frac{1}{\sqrt{1-x^2}}$ D) $\frac{-1}{\sqrt{1-x^2}}$
- 10) The derivative of $2\sqrt{\cot x^2}$ with respect to x is
 A) $\frac{-2x \operatorname{cosec}^2 x^2}{\sqrt{\cot x^2}}$ B) $\frac{-\operatorname{cosec} x^2}{\sqrt{\cot x^2}}$ C) $\frac{2x \operatorname{cosec}^2 x^2}{\sqrt{\cot x^2}}$ D) $\frac{-2x \operatorname{cosec} x}{\sqrt{\cot x^2}}$
- 11) The function $f(x) = |x + 1| + |x - 1|$ is
 A) Continuous everywhere but not differentiable at exactly one point
 B) Continuous everywhere but not differentiable at exactly two points
 C) Continuous everywhere but not differentiable at exactly three points
 D) Continuous everywhere and differentiable everywhere
- 12) The total revenue in rupees received from the sale of x units of a product is given by $R(x) = 13x^2 + 26x - 15$, then marginal revenue when $x = 1$ is
 A) ₹ 126 B) ₹ 13 C) ₹ 52 D) ₹ 104
- 13) Assertion (A) : The maximum value of the function $f(x) = x^3, \forall x \in [-1, 1]$ is attained at its end point $x = 1$.
 Reason (R) : The function $f(x) = x^3$ is strictly increasing function $[-1, 1]$
 A) (A) is false but (R) is true
 B) (A) is false but (R) is false
 C) Both (A) and (R) are true and (R) is the correct explanation for (A)
 D) Both (A) and (R) are true and (R) is not a correct explanation for (A)
- 14) The value of $\int \left(\frac{x^3 - x^2 + x - 1}{x - 1} \right) dx$ is equal to
 A) $\frac{x^3}{3} + x + C$ B) $x^2 + x + C$ C) $\frac{x^3}{3} - x + C$ D) $\frac{x^3}{3} + \frac{x^2}{2} + x + C$
- 15) The antiderivative of the function $\frac{1 - \sin x}{\cos^2 x}$ with respect with x is
 A) $\sec x + \tan x + C$ B) $\sec x - \tan x + C$ C) $-\tan x - \sec x + C$ D) $\tan x - \sec x + C$
- II Fill in the blanks by choosing the appropriate answer from those given in the bracket:**
 [11, 2, 4, 0, 1.4, -1] 5x1=5
- 16) If A and B are square matrices of same order such that $|AB| = 16, |A| = 8$ then $|B| = \underline{\hspace{2cm}}$.
- 17) The value of $\tan^2(\sec^{-1} 2) + \cot^2(\operatorname{cosec}^{-1} 3)$ is equal to .
- 18) If $y = 2\cos x + 3 \sin x$, then $\frac{d^2y}{dx^2} + y = \underline{\hspace{2cm}}$.
- 19) The minimum value of the function $f(x) = |x + 2| - 1$ is .
- 20) The radius of the circle is increasing at the rate of 0.7 cm/sec, then the rate of increase of its circumference is π cm / sec.

PART -B

III Answer any SIX questions :

- 21) Show that the function $f: N \rightarrow N$ given by $f(1) = f(2) = 1$ and $f(x) = x - 1$, for every $x > 2$, is onto but not one-one. 6x2=12

- 22) Prove that $3 \cos^{-1} x = \cos^{-1} (4x^3 - 3x)$, $\forall x \in \left[\frac{1}{2}, 1\right]$.
- 23) Prove that $\sin^{-1}\left(\frac{8}{17}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{77}{36}\right)$.
- 24) Find the value of k, if area of the triangle is 4 sq. units and the vertices are (k, 0), (4, 0) and (0, 2) using determinants.
- 25) If $A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, then find $(A + 2B)'$.
- 26) If $y = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, then prove that $\frac{dy}{dx} = \frac{2}{1+x^2}$.
- 27) Find the intervals in which the function f given $f(x) = 2x^2 - 3x$ is strictly increasing.
- 28) Evaluate : $\int \sin 2x \cos 3x \, dx$.
- 29) Evaluate : $\int \frac{1}{x + x \log x} \, dx$.

PART-C

V Answer any SIX of the following questions :

6x3=18

- 30) Show that the relation R defined in the set of integers given by $R = \{(a, b) / 2 \text{ divides } a-b\}$ is an equivalence relation.
- 31) Express $\tan \left\{ \frac{1}{2} \left[\sin^{-1} \left(\frac{2x}{1+x^2} \right) + \cos^{-1} \left(\frac{1-y^2}{1+y^2} \right) \right] \right\}$, $|x| < 1, y > 0$ and $xy < 1$, in the simplest form.
- 32) Express the matrix $\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$ as sum of symmetric and skew symmetric matrix.
- 33) If $y^x = x^y$, then find $\frac{dy}{dx}$.
- 34) If $x = a(\theta + \sin \theta)$ and $y = a(1 - \cos \theta)$, then prove that $\frac{dy}{dx} = \tan \left(\frac{\theta}{2} \right)$.
- 35) A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 cm/sec. How fast in its height on the wall decreasing when the foot of the ladder is 4 m away from the wall.
- 36) Find the two positive numbers x and y such that $x + y = 60$ and xy^3 is maximum.
- 37) Evaluate : $\int \frac{1}{1 - \tan x} \, dx$.
- 38) Evaluate : $\int \frac{x}{(x+1)(x+2)} \, dx$.

PART-D

4x5=20

VI Answer any FOUR questions :

- 39) Consider $f : \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = 10x + 7$. Show that f is invertible and also find its inverse.

(P.T.O.)

40) If $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 0 & 3 \\ 3 & -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ 0 & 2 \\ -1 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 & 3 & -4 \\ 2 & 0 & -2 & 1 \end{bmatrix}$, then show that $(AB)C = A(BC)$.

41) If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1} . Using A^{-1} solve the system of equation $2x - 3y + 5z = 11$,

$$3x + 2y - 4z = -5, x + y - 2z = -3$$

42) If $y = Ae^{mx} + Be^{nx}$, then prove that $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$.

43) If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, x and y are distinct, then prove that $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$.

44) Find the integral of $\frac{1}{x^2 - a^2}$ with respect to x and hence evaluate $\int \frac{1}{x^2 - 16} dx$.

45) Find the antiderivative of $\frac{1}{\sqrt{a^2 + x^2}}$ with respect to x and hence evaluate $\int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}} dx$.

PART-E

VII Answer the following questions :

(6 + 4=10)

46) Solve the following graphically : Minimize and Maximize $z = 3x + 9y$ subject to the constraints $x + 3y \leq 60$, $x \leq y$, $x + y \geq 20$, $x, y \geq 0$.

OR

Solve the following graphically : Maximize and Minimize $Z = 5x + 3y$ subject to the constraints; $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x \geq 0$, $y \geq 0$. **(6)**

47) Show that the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ satisfies the equation $A^2 - 4A + I = O$, where O is null matrix

and hence find A^{-1} .

OR

Find the value of k , if $f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x} & \text{if } x \neq \frac{\pi}{2} \\ 3 & \text{if } x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$. **(4)**
